Topic 3 : Borda's method: A Scoring System

In the plurality and runoff methods discussed in the previous lecture, we do not take into account the voter's relative preferences for *all* of the candidates. We do not, for example take into account which candidate was ranked last by each voter.

Borda's Method

Lets assume that each judge or voter ranks the candidates in order of preference. The Borda count determines the outcome of a competition or the winner of an election by giving each candidate, for each ballot, a number of points corresponding to the number of candidates ranked lower(or equal).

Simple Borda Count If **voters rank the entire list of candidates** or choices in order of preference from the first choice to the last choice then using **Borda's method** the votes are tallied as follows:

- On a particular ballot, the lowest ranking candidate is given 1 point, the second lowest is given 2 points, and so on, the top candidate receiving points equal to the number of candidates .
- The number of points given to each candidate is summed across all ballots. This is called the **Borda Count** for the candidate.
- The winner is the candidate with the highest Borda count.

A Simple Borda Count results in the same ranking as that derived from average rankings: As it turns out the ranking we get for the candidates from a Borda Count is the same as the ranking we get from the ratings calculated by averaging the votes, where a lower rating is given a better(lower) ranking.

Example In a survey, squash players were asked to rank brands of squash racquets. The results are shown below:

# Players →	**33**	**3**	**10**	**20**	**7**	**27**
Dunlop	1	1	2	3	2	3
Black Knight	2	3	1	1	3	2
Prince	3	2	3	2	1	1

(a) Which brand would win using Borda's method?

(b) Which brand would win using the Plurality method?

(c) Which brand would win using the Plurality method with a runoff between the first and second place finishers?

Example Lets apply this method to the results of our votes on video competition:

Number of Voters	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	1	1	1	2	1	1	1	1	1	1	1	1	1	2	1	1	2
Red hot Chilli Peppers	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5
Jurassic Park	2	2	2	2	3	3	4	5	5	5	5	1	3	3	5	5	1	1	2	2	4	5	5	5	1	3	3	5	5	5	1	2	4	4	4
Donnie Trumpet	3	3	4	5	4	5	5	2	2	3	4	3	4	5	1	4	2	5	4	5	5	2	4	4	3	1	2	1	1	3	4	1	1	2	3
Beyonce	4	5	3	3	5	2	2	3	4	2	3	4	1	1	3	1	4	4	1	1	1	1	1	2	2	2	1	2	3	1	3	3	3	3	1
Drake	5	4	5	4	2	4	3	4	3	4	2	5	5	4	4	3	5	2	5	4	2	4	2	1	5	5	5	3	2	2	2	4	2	1	2

Modified Borda Count: If there are N candidates and some voters do not rank the entire list of candidates, instead ranking their top M candidates, where $M < N$ we can modify the Borda count in the following way:

- On a particular ballot where the candidates are ranked 1 through M, the lowest ranking candidate is given 1 point, the second lowest is given 2 points, and so on, the top candidate receiving points equal to M . The unranked candidates are given 0 points.
- The number of points given to each caniddate is summed across all ballots. This is called the **Borda Count** for the candidate.
- The winner is the candidate with the highest Borda count.

The **ranking resulting from a modified Borda Count** for an election with N candidates is equivalent to the **ranking resulting from an averaging process** modified as follows:

- On a particular ballot where the candidates are ranked 1 through M and $M < N$, the unranked individuals all receive a rank of M + 1.
- the average across all ballots is then taken for each candidate.

Example Suppose, we had our 10 judges rank the movies nominated for best picture in the Oscars. Now perhaps the judges had not seen all of the movies and everyone just ranked the ones they had seen and the results were as follows:

# Voters	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**1**
The Big Short	4	5	8	3	3	3	1	2	
Bridge of Spies	1	3	7	2	4	2		1	2
Brooklyn	3	2	6	1			3	3	1
Mad Max: Fury Road	2	1	5	4	2	1	2		
Bring Him Home			4	5					4
The Revenant	5		3	6	1		4		5
Room		4	2	7					6
Spotlight			1	8					3

Replace the empty spaces with the appropriate numbers, calculate averages and find the resulting rankings.

Advantages, Disadvantages

The **advantages** of Borda's method (average) over plurality methods is

1. that voters are able to express their opinions about candidates other than just their first choice. This means that a candidate who is ranked highly but not necessarily first by many voters has a good chance of winning when using Borda's method.

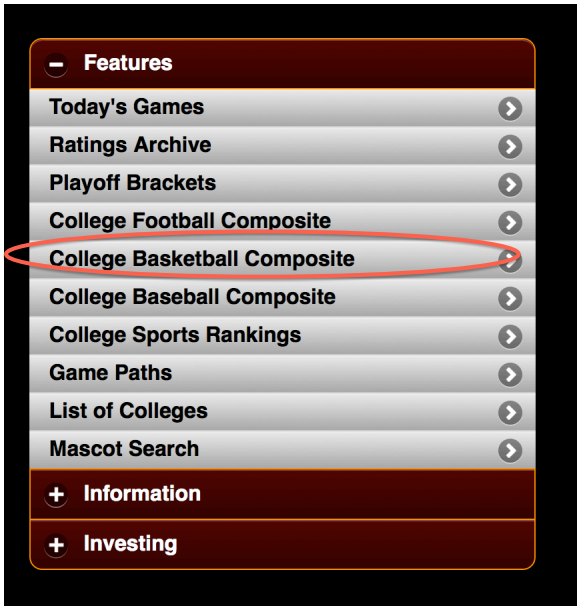
2. The average vote for a large amount of data can be computed easily on a computer.

The **disadvantage** of using Borda's method (average) is

1. that it is more susceptible to strategic voting than either the Plurality or Runoff Plurality methods (it is very easy to increase the average vote of a candidate you do not like).

Using the Computer: Since we just need to fill in blanks and calculate averages, we can use Excel to work with data pretty easily in this case. We will download some rankings relating to the NCAA mens basketball tournament and calculate the averages with Excel. We can then sort the file with the averages in increasing order to get our own ranking.

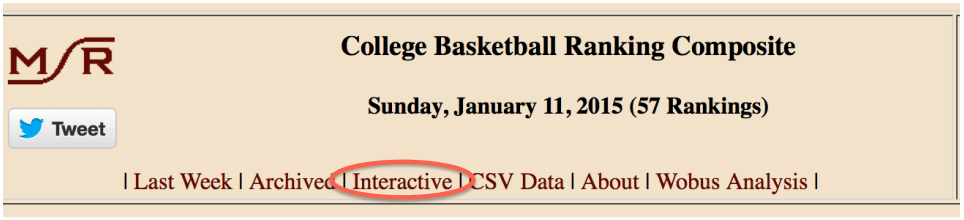
To get some data, we visit Massey's webpage: http://www.masseyratings.com/
click on College Basketball Composite in the panel on the right



This brings you to a page with current rankings for College Basketball:
http://www.masseyratings.com/cb/compare.htm

This page shows 57 current rankings(updated each Sunday during the season), the full names of which are shown across the top of the page. It also shows the average, median and standard deviation of the rankings at the end. We will study two types of computer rankings in greater detail in later lectures, namely the Colley and Massey rankings. As you can see, they give different results.

Personally from here I like to go to the interactive page by clicking on `interactive` in the panel above the rankings:



http://www.masseyratings.com/cb/comp.htm

On this page, you can choose a particular ranking by clicking on the top of the associated column and the teams will be sorted accordingly.

We can now download the data by clicking on: `CSV Data` at the top of the page:

To **create an average rank using only your favorite rankings**, you need to use the average function in Excel. If you want to use AP rankings or the Coaches Poll, which rank the top 25, you will have to fill in the blank spaces in those columns with 26's.

Example What is the ranking resulting from a Borda count (average rank) using the following polls: AP, Coaches Poll(USA), Sagarin(SAG), Massey(MAS), Colley(COL) and Pomeroy(POM) ?

Open the Excel File, you can delete the material above the rankings, being careful to keep your ranking data together by deleting the entire rows.

You can also delete or hide the columns containing the rankings you do not want.

Name your **new column** Borda or Average. Click on the cell in that column next to the first team (In Row 2 say). Type in the **formula** `= AVERAGE(K2,T2,AN2,AT2,AZ2,BJ2)` if your preferred rankings are in columns K, T, AN, AT, AZ and BJ. Then press return and the average rank should appear in that cell. Now click on the cell and **copy**, then click on the next cell in the column **and paste**, Excel will adjust the rows in the formula accordingly. When you have pasted a few, you can copy a bunch of cells together and paste them to save time.

Finally, select all of the data including the team names and choose `sort` from the menu bar on your computer. Use the column with your average rankings as the sort column and **sort** from smallest to largest. The team names should now be arranged in the ranking order corresponding to your Borda Count.

If you actually want to write down the **rank of each team** for your ranking, you can create a new column called `MYRANK`. While your teams are ordered by your Borda Count, choose the first cell in your column `MYRANK` which is in the same row as your top team. If this is Row 1, type `=ROW()` to identify the Row number and rank, If it is in Row 2 (The column heading may be in Row 1), type `=ROW()-1` to get the rank. Copy and paste your formula to the other cells in the column. Now to erase the formula (It will change the numbers if you move the rows around), you can click on the column and copy it using `CTRL-c`. Then press `Shift` and `F10` together followed by `CTRL-v` to paste the numbers back in without the formulas.

Results for my selected rankings on Jan 20 2016 at 5:40 p.m.

Team	AP	COL	MAS	POM	SAG	USA	BordaME		
Oklahoma	1	2	1	4	3	1	2	1	
Villanova	4	3	2	1	1	4	2.5	2	
Kansas	3	5	4	2	2	3	3.16666667	3	
North Carolina	2	6	8	7	6	2	5.16666667	4	
Xavier	5	1	3	8	9	6	5.33333333	5	
West Virginia	6	10	6	5	4	7	6.33333333	6	
Iowa	9	8	7	3	5	9	6.83333333	7	
Maryland	7	12	9	14	14	5	10.1666667	8	
Texas A&M	10	7	11	13	13	8	10.3333333	9	
SMU	8	4	5	12	8	26	10.5	10	
Michigan St	11	13	10	11	10	10	10.8333333	11	
Virginia	13	23	13	9	12	13	13.8333333	12	
Louisville	17	27	14	6	7	16	14.5	13	
Arizona	12	22	12	16	15	11	14.6666667	14	
Miami FL	15	18	18	15	16	14	16	15	
Purdue	22	17	15	10	11	22	16.1666667	16	
Duke	20	21	22	21	17	12	18.8333333	17	
Baylor	13	28	16	20	22	15	19	18	
Iowa St	19	19	21	19	19	21	19.6666667	19	
USC	21	15	17	22	25	25	20.8333333	20	
Pittsburgh	26	14	20	28	20	20	21.3333333	21	
Indiana	25	29	24	26	18	23	24.1666667	22	
Oregon	26	16	26	24	30	26	24.6666667	23	
St Mary's CA	26	34	25	17	21	26	24.8333333	24	
South Carolina	24	11	19	42	38	18	25.3333333	25	
Butler	18	31	29	36	23	24	26.8333333	26	
Valparaiso	26	30	28	18	33	26	26.8333333	27	
Kentucky	23	26	32	31	32	19	27.1666667	28	
Providence	16	20	23	46	43	17	27.5	29	
Dayton	26	9	33	45	36	26	29.1666667	30	
Notre Dame	26	44	27	25	27	26	29.1666667	31	
Michigan	26	42	30	30	24	26	29.6666667	32	
Florida	26	41	39	27	29	26	31.3333333	33	
Wichita St	26	47	35	23	31	26	31.3333333	34	
Gonzaga	26	40	42	39	26	26	33.1666667	35	
Colorado	26	25	34	48	41	26	33.3333333	36	
Florida St	26	50	31	38	39	26	35	37	
Seton Hall	26	37	37	44	47	26	36.1666667	38	
Evansville	26	49	38	35	48	26	37	39	
St Joseph's PA	26	24	40	54	54	26	37.3333333	40	
VA Commonw	26	56	43	33	40	26	37.3333333	41	
Connecticut	26	67	44	34	35	26	38.6666667	42	
Utah	26	33	36	66	46	26	38.8333333	43	
California	26	57	51	37	37	26	39	44	
Texas	26	38	49	53	42	26	39	45	
Cincinnati	26	72	46	32	34	26	39.3333333	46	
Monmouth NJ	26	43	45	43	56	26	39.8333333	47	
Vanderbilt	26	75	60	29	28	26	40.6666667	48	
Texas Tech	26	36	54	51	52	26	40.8333333	49	
Ark Little Rock	26	32	41	50	77	26	42	50	

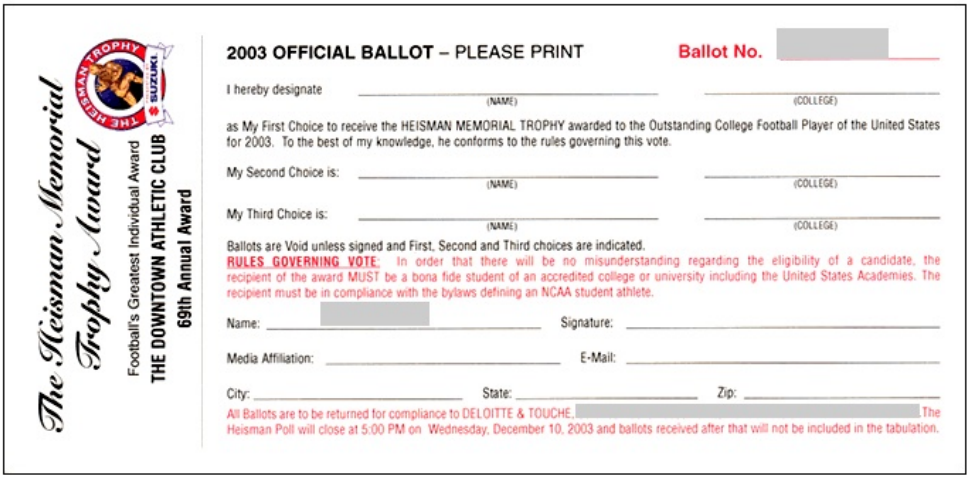
Variations of The Borda Count in Sports

In sports polls where this form of voting is commonly used, the voters may know a lot about the top teams or players and be able to rank them, but may not know enough to rank all eligible candidates, so lumping all but the top candidates together with 0 points simplifies the process for voters.

Heisman Trophy

A variation of this method is used to decide the winner of the Heisman Trophy. On the Heisman ballot voters are asked to **rank only their top three choices from among all college football players in the United States**. The Borda count for each player is computed by giving 3 points for each first place vote, 2 points for each second place vote and 1 point for each third place vote. The winner is declared to be the candidate with the highest Borda count.

THE 2003 OFFICIAL HEISMAN BALLOT



The Heisman Memorial Trophy Award
Football's Greatest Individual Award
THE DOWNTOWN ATHLETIC CLUB
69th Annual Award

2003 OFFICIAL BALLOT – PLEASE PRINT Ballot No. [redacted]

I hereby designate _____ (NAME) _____ (COLLEGE)

as My First Choice to receive the HEISMAN MEMORIAL TROPHY awarded to the Outstanding College Football Player of the United States for 2003. To the best of my knowledge, he conforms to the rules governing this vote.

My Second Choice is: _____ (NAME) _____ (COLLEGE)

My Third Choice is: _____ (NAME) _____ (COLLEGE)

Ballots are Void unless signed and First, Second and Third choices are indicated.

RULES GOVERNING VOTE: In order that there will be no misunderstanding regarding the eligibility of a candidate, the recipient of the award MUST be a bona fide student of an accredited college or university including the United States Academies. The recipient must be in compliance with the bylaws defining an NCAA student athlete.

Name: [redacted] Signature: _____

Media Affiliation: _____ E-Mail: _____

City: _____ State: _____ Zip: _____

All Ballots are to be returned for compliance to DELOITTE & TOUCHE [redacted] The Heisman Poll will close at 5:00 PM on Wednesday, December 10, 2003 and ballots received after that will not be included in the tabulation.

NBA most valuable player (uses wider spread of points)

To decide on the winner of the National Basketball Association Most Valuable Player award, **116 members of the media list their first through fifth choices for the award.** Each first place vote receives 10 points, each second place vote receives 7 points, each third place vote receives 5 points, each fourth place vote receives 3 points and each fifth place vote receives 1 point.

AP Polls

The Associated Press (AP) college football poll gives a ranking for the top teams in college football. The voters in the AP poll are newspaper, radio and television sports reporters throughout the country. In 2011 there were 60 voters and the final rankings are given in the table below.

AP Top 25

RK	TEAM	RECORD	PTS	PVS
1	Alabama (55)	12-1	1495	2
2	LSU (1)	13-1	1425	1
3	Oklahoma State (4)	12-1	1399	3
4	Oregon	12-2	1250	6
5	Arkansas	11-2	1198	7
6	USC	10-2	1181	5
7	Stanford	11-2	1167	4
8	Boise State	12-1	1127	8
9	South Carolina	11-2	1013	10
10	Wisconsin	11-3	905	9
11	Michigan State	11-3	873	12
12	Michigan	11-2	839	13
13	Baylor	10-3	780	15
14	TCU	11-2	653	16
15	Kansas State	10-3	621	11
16	Oklahoma	10-3	572	19
17	West Virginia	10-3	547	23
18	Houston	13-1	518	20
19	Georgia	10-4	439	18
20	Southern Miss	12-2	411	22
21	Virginia Tech	11-3	329	17
22	Clemson	10-4	188	14
23	Florida State	9-4	154	25
24	Nebraska	9-4	143	21
25	Cincinnati	10-3	103	NR

Others receiving votes: Brigham Young 51, Auburn 40, Northern Illinois 33, Missouri 23, Texas 15, Rutgers 3, North Dakota State 2, Penn State 2, Virginia 1